Phase transition and critical behavior of d=3 chiral fermion models with left/right asymmetry

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We investigate the critical behavior of three-dimensional relativistic fermion models with a $U(N_L)_L \times U(1)_R$ chiral symmetry reminiscent of the Higgs-Yukawa sector of the standard model of particle physics. We classify all possible four-fermion interaction terms and the corresponding discrete symmetries. For sufficiently strong correlations in a scalar parity-conserving channel, the system can undergo a second-order phase transition to a chiral-symmetry broken phase which is a 3d analog of the electroweak phase transition. We determine the critical behavior of this phase transition in terms of the critical exponent ν and the fermion and scalar anomalous dimensions for $N_L \geq 1$. Our models define new universality classes that can serve as prototypes for studies of strongly correlated chiral fermions.

I. INTRODUCTION

The interplay between statistical physics and particle physics has stimulated substantial progress in both areas in the last decades. It has given rise to the renormalization group (RG) [1, 2, 3] which has led to a deep understanding of critical phenomena on the statistical-physics side and of the possible structure of fundamental interactions on the particle-physics side. In most cases, this interplay arises from analogies on the technical level of dealing with fluctuations either of statistical or quantum nature.

In the present work, we intend to push this interplay one step further: we aim at a construction of a statistical-physics model (or a field-theoretic variant thereof) which has structural similarities to a crucial building block of the standard model of particle physics: the Higgs-Yukawa sector. We are motivated by the fact that a profound understanding of electroweak symmetry breaking in the standard model or alternative scenarios may require a quantitative control of fluctuating chiral fermions and bosons beyond perturbation theory.[1] This is a challenging problem which is even lacking a simpler benchmark example where nonperturbative techniques can prove their validity.

A stage for quantitative comparisons between field-theoretical tools is set by critical phenomena of lower-dimensional systems, most prominently the computation of critical exponents of second-order phase transitions. In fact, relativistic fermionic models such as the Gross-Neveu model in 3d are known to exhibit a second-order phase transition; corresponding studies of the critical behavior have been performed by various methods [6, 7].

Near the phase transition, the critical behavior is determined by fluctuations of fermions as well as bosonic bi-fermion composites; the expectation value of the latter also serves as an order parameter for the phase transition.

In the same spirit, we devote this work to a construction of chiral four-fermion models with a $U(N_L)_L \times U(1)_R$ symmetry. Such models support a left/right asymmetry similar to the chiral structure of the Higgs-Yukawa sector. If such models undergo a phase transition to an ordered phase with broken chiral symmetry this transition can be viewed as an analogue of the electroweak phase transition in 3d. The corresponding critical behavior defines new universality classes, the properties of which are a pure prediction of the system.

In the past, $d = 3$ dimensional fermionic systems with left/right symmetric chiral symmetries such as QED$_3$ or the Thirring model have been under investigation in a variety of scenarios [8, 9, 10, 11, 12, 13] with applications to condensed-matter physics, high-T_c cuprate superconductors [14] and, recently, graphene [15]. In some of these models, the number of fermion flavors serves as a control parameter for a quantum phase transition. As the critical number of fermions is an important quantity, nonperturbative information about these models for varying flavor number N_f is required. Since good chiral fermion properties for arbitrary N_f for massless fermions still represents a challenge, e.g., for lattice simulations, other powerful nonperturbative techniques are urgently needed.

In this work, we investigate this class of chirally symmetric fermion models with left/right asymmetry by means of the functional RG. The functional RG and associated RG flow equations such as the Wetterich equation [16] are an appropriate tool for this purpose, since fermions and bosons can both be treated equally well. Also, the flow equation is an exact equation and facilitates the construction of non-perturbative approximation schemes, see [17] for reviews.

After a classification of possible fermionic models in Sec. II, we concentrate on strong correlations in a scalar parity-conserving channel. We study the critical behavior of a possible condensation of bosonic bi-fermion com-

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[1] This work has partly been inspired by a recently discovered asymptotic-safety scenario for the Higgs-Yukawa sector of the standard model which aims at a solution of the triviality and an improvement of the hierarchy problem of the standard-model Higgs sector [4, 5].

posites into this channel. The resulting effective scalar-fermion Yukawa model is reminiscent to the standard-model Higgs-Yukawa sector. We analyze this effective Yukawa model in Sec. III with the aid of the functional RG, deriving the nonperturbative RG flow equations to next-to-leading order in a derivative expansion.

The left/right asymmetry of our model is controlled by varying the number N_L of left-handed fermions. This imbalance provides us with an external control parameter for the relative amplitude of boson and fermion fluctuations. This allows us to vary the fixed-point properties of the RG flow, implying also a variation of the critical properties of these systems. In particular, the fixed-point potential for the bosonic order parameter can be in the symmetric regime for small N_L or in the broken regime for larger N_L, as discussed in Sec. IV. The small N_L regime turns out to be particularly interesting, as the anomalous dimensions of all fields have to satisfy a sum rule within our truncation, in order to give rise to a nontrivial fixed point in the Yukawa coupling.

The quantitative reliability of our results for the critical properties can be checked by a reduction of our model to a corresponding purely bosonic O(N) model. For the latter, our method has exhaustively been investigated and used with a remarkable quantitative success [17, 18, 19]; the results for critical exponents have reached an accuracy which is comparable if not superior to that of, e.g., lattice simulations and high-order ϵ expansions, see [19] for a recent comparison. Our inclusion of fermions can reliably be based on this footing, as our models approach an O($2N_L$) model in the limit of large N_L; this is because the composite scalar degrees of freedom dominate the fluctuation contributions as a result of the chiral structure. For smaller N_L, our results show quantitative as well as qualitative differences to O(N) models, as expected due to the fermionic contributions. The resulting models and quantitative findings constitute and characterize a new set of universality classes, classified by the chiral symmetry content. In particular, we provide for quantitative predictions for the critical exponents for these universality classes, which have not been investigated with any other method so far. We believe that these can serve as a first benchmark for other nonperturbative methods which are urgently needed for a study of chiral phase transitions in strongly correlated chiral fermions.

II. CLASSICAL ACTION AND SYMMETRY TRANSFORMATIONS

Let us first consider a general fermionic model in $d = 2 + 1$ Euclidean dimensions with local quartic self-interaction, being invariant under chiral U(N_L)$_L \otimes$ U(N_R)$_R$ transformations. The Dirac algebra

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}, \tag{1}$$

could minimally be realized by an irreducible representation in terms of 2×2 matrices. As this representation does not permit a chiral symmetry, the massless theory could not be separated from the massive theory by an order-disorder transition. We therefore work exclusively with a 4×4 *reducible* representation of the Dirac algebra

$$\gamma_\mu = \begin{pmatrix} 0 & -i\sigma_\mu \\ i\sigma_\mu & 0 \end{pmatrix}, \quad \mu = 1, 2, 3, \tag{2}$$

with $\{\sigma_1, \sigma_2, \sigma_3\}$ being the 2×2 Pauli matrices. Such models with four-component fermions in three dimensions have extensively been studied in the past (e.g. in the context of spontaneous chiral symmetry breaking in QED$_3$ or the Thirring model) [8, 9, 10]. There has been renewed interest in the last years, due to potential applications to high-T_c superconductivity in cuprates or electronic properties of graphene [11, 12, 15]. For a review, see e.g., [13]. There are now *two* other 4×4 matrices which anticommute with all γ_μ as well as with each other,

$$\gamma_4 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad \text{and} \quad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}. \tag{3}$$

Together with

$$\mathbb{1}, \; \sigma_{\mu\nu} := \frac{i}{2}[\gamma_\mu, \gamma_\nu] \; (\mu < \nu), \; i\gamma_\mu\gamma_4, \; i\gamma_\mu\gamma_5, \; i\gamma_4\gamma_5, \tag{4}$$

these 16 matrices form a complete basis of the 4×4 Dirac algebra,

$$\{\gamma_A\}_{A=1,\dots,16} = \{\mathbb{1}, \gamma_\mu, \gamma_4, \sigma_{\mu\nu}, i\gamma_\mu\gamma_4, i\gamma_\mu\gamma_5, i\gamma_4\gamma_5, \gamma_5\}. \tag{5}$$

Now, we define chiral projectors

$$P_{L/R} = \frac{1}{2}(\mathbb{1} \pm \gamma_5) \tag{6}$$

that allow us to decompose a Dirac fermion ψ into the left- and right-handed Weyl spinors ψ_L and ψ_R,

$$\psi_{L/R} = P_{L/R}\psi, \quad \bar{\psi}_{L/R} = \bar{\psi}P_{R/L}. \tag{7}$$

(ψ and $\bar{\psi}$ are considered as independent field variables in our Euclidean formulation.) Note that there is a certain freedom of choice of the notion of chirality here: we could have chosen just as well $\tilde{P}_{L/R} = (\mathbb{1} \pm \gamma_4)/2$ or $\hat{P}_{L/R} = (\mathbb{1} \pm i\gamma_4\gamma_5)/2$ as chiral projectors. This would have led us to different definitions of the decomposition into Weyl spinors. All these chiralities remain conserved under Lorentz transformations since all three projectors commute with the generators of the Lorentz transformation of the Dirac spinors, $[\gamma_5, \sigma_{\mu\nu}] = [\gamma_4, \sigma_{\mu\nu}] = [i\gamma_4\gamma_5, \sigma_{\mu\nu}] = 0$.

We consider N_R right-handed and N_L left-handed fermions, where N_R and N_L do not have to be identical. We impose a chiral U(N_L)$_L \otimes$ U(N_R)$_R$ symmetry with corresponding field transformations which act independently on left- and right-handed spinors,

$$\text{U}(N_L)_L: \quad \psi_L^a \mapsto U_L^{ab}\psi_L^b, \quad \bar{\psi}_L^a \mapsto \bar{\psi}_L^b(U_L^\dagger)^{ba}, \tag{8}$$

$$\text{U}(N_R)_R: \quad \psi_R^a \mapsto U_R^{ab}\psi_R^b, \quad \bar{\psi}_R^a \mapsto \bar{\psi}_R^b(U_R^\dagger)^{ba}. \tag{9}$$

Here, U_L and U_R are unitary $N_\mathrm{L} \times N_\mathrm{L}$ and $N_\mathrm{R} \times N_\mathrm{R}$ matrices, respectively. For $U_\mathrm{L}^{ab} = \mathrm{e}^{\mathrm{i}\alpha}\,\delta^{ab}$ and $U_\mathrm{R}^{ab} = \mathrm{e}^{-\mathrm{i}\alpha}\,\delta^{ab}$ we obtain the usual $\mathrm{U}(1)_\mathrm{A}$ axial transformations, whereas for $U_\mathrm{L}^{ab} = \mathrm{e}^{\mathrm{i}\alpha}\,\delta^{ab}$ and $U_\mathrm{R}^{ab} = \mathrm{e}^{\mathrm{i}\alpha}\,\delta^{ab}$ we get $\mathrm{U}(1)_\mathrm{V}$ phase rotations. The symmetry thus is

$$\mathrm{U}(N_\mathrm{L})_\mathrm{L}\otimes\mathrm{U}(N_\mathrm{R})_\mathrm{R} \cong \mathrm{SU}(N_\mathrm{L})_\mathrm{L}\otimes\mathrm{SU}(N_\mathrm{R})_\mathrm{R}\otimes\mathrm{U}(1)_\mathrm{A}\otimes\mathrm{U}(1)_\mathrm{V}, \tag{10}$$

with chiral $\mathrm{SU}(N_\mathrm{L,R})$ factors.

Due to the reducible representation of the Dirac algebra, there is also some freedom in the definition of the discrete transformations [10]. Charge conjugation may be implemented by either

$$\mathcal{C}:\ \psi_\mathrm{L/R}^a \mapsto \left(\bar\psi_\mathrm{L/R}^a C\right)^T,\ \bar\psi_\mathrm{L/R}^a \mapsto -\left(C^\dagger \psi_\mathrm{L/R}^a\right)^T, \tag{11}$$

with $C = \gamma_2\gamma_5$, or

$$\tilde{\mathcal{C}}:\ \psi_\mathrm{L/R}^a \mapsto \left(\bar\psi_\mathrm{R/L}^a \tilde{C}\right)^T,\ \bar\psi_\mathrm{L/R}^a \mapsto -\left(\tilde{C}^\dagger \psi_\mathrm{R/L}^a\right)^T, \tag{12}$$

with $\tilde{C} = \gamma_2\gamma_4$, or a unitary combination thereof. In the same manner, the parity transformation corresponding to

$$(x_1, x_2, x_3) \mapsto (-x_1, x_2, x_3) =: \tilde{x}, \tag{13}$$

with (x_1, x_2) as space coordinates and x_3 as the (Euclidean) time coordinate, may be implemented by either

$$\mathcal{P}:\ \psi_\mathrm{L/R}^a(x) \mapsto P\psi_\mathrm{L/R}^a(\tilde{x}),\ \bar\psi_\mathrm{L/R}^a(x) \mapsto \bar\psi_\mathrm{L/R}^a(\tilde{x})P^\dagger, \tag{14}$$

with $P = \gamma_1\gamma_4$, or

$$\tilde{\mathcal{P}}:\ \psi_\mathrm{L/R}^a(x) \mapsto \tilde{P}\psi_\mathrm{R/L}^a(\tilde{x}),\ \bar\psi_\mathrm{L/R}^a(x) \mapsto \bar\psi_\mathrm{R/L}^a(\tilde{x})\tilde{P}^\dagger, \tag{15}$$

with $\tilde{P} = \gamma_1\gamma_5$. Similarly, time reversal corresponding to

$$(x_1, x_2, x_3) \mapsto (x_1, x_2, -x_3) =: \hat{x} \tag{16}$$

reads either

$$\mathcal{T}:\ \psi_\mathrm{L/R}^a(x) \mapsto T\psi_\mathrm{L/R}^a(\hat{x}),\ \bar\psi_\mathrm{L/R}^a(x) \mapsto \bar\psi_\mathrm{L/R}^a(\hat{x})T^\dagger, \tag{17}$$

with $T = \gamma_2\gamma_3$, or

$$\tilde{\mathcal{T}}:\ \psi_\mathrm{L/R}^a(x) \mapsto \tilde{T}\psi_\mathrm{R/L}^a(\hat{x}),\ \bar\psi_\mathrm{L/R}^a(x) \mapsto \bar\psi_\mathrm{R/L}^a(\hat{x})\tilde{T}^\dagger, \tag{18}$$

with $\tilde{T} = \gamma_1$. Note that every matrix $\in \{C, \tilde{C}, P, \tilde{P}, T, \tilde{T}\}$ is unitary, such that charge conjugation and parity inversion are unitary, and time reversal is anti-unitary.

In order to derive the explicit transformation properties of the bilinears, it is useful to recall that γ_1 and γ_3 are antisymmetric and purely imaginary, whereas γ_2, γ_4, and γ_5 are symmetric and real. The results are listed in Table I, where we have introduced

$$\tilde\gamma_\mu := (-\gamma_1, \gamma_2, \gamma_3)_\mu, \tag{19}$$
$$(\tilde\sigma_{12}, \tilde\sigma_{13}, \tilde\sigma_{23}) := (-\sigma_{12}, -\sigma_{13}, \sigma_{23}), \tag{20}$$
$$\hat\gamma_\mu := (\gamma_1, \gamma_2, -\gamma_3)_\mu, \tag{21}$$
$$(\hat\sigma_{12}, \hat\sigma_{13}, \hat\sigma_{23}) := (\sigma_{12}, -\sigma_{13}, -\sigma_{23}). \tag{22}$$

These transformation properties facilitate a discussion of possible bilinears and 4-fermi terms in the action of our model. In addition to Lorentz invariance, we impose an invariance of our theory under $\mathrm{U}(N_\mathrm{L})_\mathrm{L} \otimes \mathrm{U}(N_\mathrm{R})_\mathrm{R}$ chiral transformations, $\mathcal{C}$ charge conjugation, $\mathcal{P}$ parity inversion, and $\mathcal{T}$ time reversal. The theory then automatically is also invariant under $\tilde{\mathcal{C}}\tilde{\mathcal{P}}$, $\tilde{\mathcal{P}}\tilde{\mathcal{T}}$, and $\tilde{\mathcal{C}}\tilde{\mathcal{T}}$ transformations, since $\tilde{\mathcal{C}}\tilde{\mathcal{P}} = \mathcal{C}\mathcal{P}$, $\tilde{\mathcal{P}}\tilde{\mathcal{T}} = \mathcal{P}\mathcal{T}$, and $\tilde{\mathcal{C}}\tilde{\mathcal{T}} = \mathcal{C}\mathcal{T}$. (The equivalence holds up to $\mathrm{U}(1)_\mathrm{V}$ phase rotations of the spinors.) As a consequence, no bilinears to zeroth order in derivatives are permitted. To first order, only the standard chiral kinetic terms

$$\bar\psi_\mathrm{L}^a \mathrm{i}\partial_\mu \gamma_\mu \psi_\mathrm{L}^a \quad\text{and}\quad \bar\psi_\mathrm{R}^a \mathrm{i}\partial_\mu \gamma_\mu \psi_\mathrm{R}^a \tag{23}$$

can appear. In particular, all possible mass terms are excluded by symmetry: $\bar\psi_\mathrm{L}^a \psi_\mathrm{R}^a$ and $\bar\psi_\mathrm{R}^a \psi_\mathrm{L}^a$ are not chirally symmetric, and $\bar\psi_\mathrm{L}^a \gamma_4 \psi_\mathrm{L}^a$ as well as $\bar\psi_\mathrm{R}^a \gamma_4 \psi_\mathrm{R}^a$ are not invariant under $\mathcal{P}$ parity transformations. The same holds for terms involving $\mathrm{i}\gamma_4\gamma_5$. On the level of 4-fermi operators, the interaction terms must have the form

$$\left(\bar\psi_\mathrm{L}^a \gamma_A \psi_\mathrm{L}^b\right)\left(\bar\psi_\mathrm{L}^b \gamma_A \psi_\mathrm{L}^a\right), \quad \left(\bar\psi_\mathrm{R}^a \gamma_A \psi_\mathrm{R}^b\right)\left(\bar\psi_\mathrm{R}^b \gamma_A \psi_\mathrm{R}^a\right) \tag{24}$$

with $\gamma_A \in \{\gamma_\mu, \gamma_4\}$,

$$\left(\bar\psi_\mathrm{L}^a \gamma_B \psi_\mathrm{R}^b\right)\left(\bar\psi_\mathrm{R}^b \gamma_B \psi_\mathrm{L}^a\right) \quad\text{with}\quad \gamma_B \in \{\mathbb{1}, \mathrm{i}\gamma_\mu\gamma_4\}, \tag{25}$$

or, with inverse flavor structure,

$$\left(\bar\psi_\mathrm{L}^a \gamma_A \psi_\mathrm{L}^a\right)\left(\bar\psi_\mathrm{L}^b \gamma_A \psi_\mathrm{L}^b\right), \quad \left(\bar\psi_\mathrm{R}^a \gamma_A \psi_\mathrm{R}^a\right)\left(\bar\psi_\mathrm{R}^b \gamma_A \psi_\mathrm{R}^b\right),$$
$$\left(\bar\psi_\mathrm{R}^a \gamma_A \psi_\mathrm{R}^a\right)\left(\bar\psi_\mathrm{L}^b \gamma_A \psi_\mathrm{L}^b\right). \tag{26}$$

Terms with $\gamma_A \in \{\mathrm{i}\gamma_\mu\gamma_5, \mathrm{i}\gamma_4\gamma_5\}$ or $\gamma_B \in \{\gamma_5, \sigma_{\mu\nu}\}$ are equal to these up to a possible sign, since ψ_L and ψ_R are eigenvectors of γ_5, and $\sigma_{\mu\nu} = -\mathrm{i}\epsilon_{\mu\nu\rho}\gamma_\rho\gamma_4\gamma_5$. Terms with $\gamma_A \in \{\mathbb{1}, \gamma_5, \mathrm{i}\gamma_\mu\gamma_4, \sigma_{\mu\nu}\}$ or $\gamma_B \in \{\gamma_\mu, \mathrm{i}\gamma_\mu\gamma_5, \gamma_4, \mathrm{i}\gamma_4\gamma_5\}$ are identically zero, since $P_\mathrm{R}P_\mathrm{L} = P_\mathrm{L}P_\mathrm{R} = 0$. The terms in Eq. (II) are not independent of the terms in Eqs. (24), (25), but are related by Fierz transformations:

$$\left(\bar\psi_\mathrm{L}^a \gamma_\mu \psi_\mathrm{L}^a\right)\left(\bar\psi_\mathrm{L}^b \gamma_\mu \psi_\mathrm{L}^b\right) = \frac{1}{2}\left(\bar\psi_\mathrm{L}^a \gamma_\mu \psi_\mathrm{L}^b\right)\left(\bar\psi_\mathrm{L}^b \gamma_\mu \psi_\mathrm{L}^a\right) + \frac{3}{2}\left(\bar\psi_\mathrm{L}^a \gamma_4 \psi_\mathrm{L}^b\right)\left(\bar\psi_\mathrm{L}^b \gamma_4 \psi_\mathrm{L}^a\right), \tag{27}$$

$$\left(\bar\psi_\mathrm{L}^a \gamma_4 \psi_\mathrm{L}^a\right)\left(\bar\psi_\mathrm{L}^b \gamma_4 \psi_\mathrm{L}^b\right) = \frac{1}{2}\left(\bar\psi_\mathrm{L}^a \gamma_\mu \psi_\mathrm{L}^b\right)\left(\bar\psi_\mathrm{L}^b \gamma_\mu \psi_\mathrm{L}^a\right) - \frac{1}{2}\left(\bar\psi_\mathrm{L}^a \gamma_4 \psi_\mathrm{L}^b\right)\left(\bar\psi_\mathrm{L}^b \gamma_4 \psi_\mathrm{L}^a\right), \tag{28}$$

TABLE I: Properties of fermion bilinears under discrete transformations. The arguments of the transformed fields are $\tilde{x} = (-x_1, x_2, x_3)$ in the case of parity and $\hat{x} = (x_1, x_2, -x_3)$ in the case of time reversal. The bilinears with (L $\leftrightarrow$ R) transform analogously.

	$\mathcal{C}$	$\tilde{\mathcal{C}}$	$\mathcal{P}$	$\tilde{\mathcal{P}}$	$\mathcal{T}$	$\tilde{\mathcal{T}}$
$\bar{\psi}_L^a \psi_R^b$	$\bar{\psi}_R^b \psi_L^a$	$\bar{\psi}_L^b \psi_R^a$	$\bar{\psi}_L^a \psi_R^b$	$\bar{\psi}_R^a \psi_L^b$	$\bar{\psi}_L^a \psi_R^b$	$\bar{\psi}_R^a \psi_L^b$
$\bar{\psi}_L^a \gamma_\mu \psi_L^b$	$-\bar{\psi}_L^b \gamma_\mu \psi_L^a$	$-\bar{\psi}_R^b \gamma_\mu \psi_R^a$	$\bar{\psi}_L^a \tilde{\gamma}_\mu \psi_L^b$	$\bar{\psi}_R^a \tilde{\gamma}_\mu \psi_R^b$	$-\bar{\psi}_L^a \hat{\gamma}_\mu \psi_L^b$	$-\bar{\psi}_R^a \hat{\gamma}_\mu \psi_R^b$
$\bar{\psi}_L^a \sigma_{\mu\nu} \psi_R^b$	$-\bar{\psi}_R^b \sigma_{\mu\nu} \psi_L^a$	$-\bar{\psi}_L^b \sigma_{\mu\nu} \psi_R^a$	$\bar{\psi}_L^a \tilde{\sigma}_{\mu\nu} \psi_R^b$	$\bar{\psi}_R^a \tilde{\sigma}_{\mu\nu} \psi_L^b$	$-\bar{\psi}_L^a \hat{\sigma}_{\mu\nu} \psi_R^b$	$-\bar{\psi}_R^a \hat{\sigma}_{\mu\nu} \psi_L^b$
$\bar{\psi}_L^a \gamma_4 \psi_L^b$	$\bar{\psi}_L^b \gamma_4 \psi_L^a$	$-\bar{\psi}_R^b \gamma_4 \psi_R^a$	$-\bar{\psi}_L^a \gamma_4 \psi_L^b$	$\bar{\psi}_R^a \gamma_4 \psi_R^b$	$\bar{\psi}_L^a \gamma_4 \psi_L^b$	$-\bar{\psi}_R^a \gamma_4 \psi_R^b$
$\bar{\psi}_L^a i\gamma_\mu \gamma_4 \psi_R^b$	$\bar{\psi}_R^b i\gamma_\mu \gamma_4 \psi_L^a$	$-\bar{\psi}_L^b i\gamma_\mu \gamma_4 \psi_R^a$	$-\bar{\psi}_L^a i\tilde{\gamma}_\mu \gamma_4 \psi_R^b$	$\bar{\psi}_R^a i\tilde{\gamma}_\mu \gamma_4 \psi_L^b$	$\bar{\psi}_L^a i\hat{\gamma}_\mu \gamma_4 \psi_R^b$	$-\bar{\psi}_R^a i\hat{\gamma}_\mu \gamma_4 \psi_L^b$

$$\left(\bar{\psi}_R^a \gamma_\mu \psi_R^a\right)\left(\bar{\psi}_L^b \gamma_\mu \psi_L^b\right) = -\frac{3}{2}\left(\bar{\psi}_R^a \psi_L^b\right)\left(\bar{\psi}_L^b \psi_R^a\right) - \frac{1}{2}\left(\bar{\psi}_R^a i\gamma_\mu \gamma_4 \psi_L^b\right)\left(\bar{\psi}_L^b i\gamma_\mu \gamma_4 \psi_R^a\right), \tag{29}$$

$$\left(\bar{\psi}_R^a \gamma_4 \psi_R^a\right)\left(\bar{\psi}_L^b \gamma_4 \psi_L^b\right) = -\frac{1}{2}\left(\bar{\psi}_R^a \psi_L^b\right)\left(\bar{\psi}_L^b \psi_R^a\right) + \frac{1}{2}\left(\bar{\psi}_R^a i\gamma_\mu \gamma_4 \psi_L^b\right)\left(\bar{\psi}_L^b i\gamma_\mu \gamma_4 \psi_R^a\right). \tag{30}$$

Here, we have suppressed the analogous equations with (L $\leftrightarrow$ R) for simplicity. We thus end up with six independent 4-fermi terms preserving $U(N_L)_L \otimes U(N_R)_R$ chiral and $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{T}$ symmetry,

$$\left(\bar{\psi}_L^a \psi_R^b\right)\left(\bar{\psi}_R^b \psi_L^a\right), \tag{31}$$

$$\left(\bar{\psi}_L^a \gamma_4 \psi_L^b\right)\left(\bar{\psi}_L^b \gamma_4 \psi_L^a\right), \quad \left(\bar{\psi}_R^a \gamma_4 \psi_R^b\right)\left(\bar{\psi}_R^b \gamma_4 \psi_R^a\right), \tag{32}$$

$$\left(\bar{\psi}_L^a \gamma_\mu \psi_L^b\right)\left(\bar{\psi}_L^b \gamma_\mu \psi_L^a\right), \quad \left(\bar{\psi}_R^a \gamma_\mu \psi_R^b\right)\left(\bar{\psi}_R^b \gamma_\mu \psi_R^a\right), \tag{33}$$

$$\left(\bar{\psi}_R^a i\gamma_\mu \gamma_4 \psi_R^b\right)\left(\bar{\psi}_R^b i\gamma_\mu \gamma_4 \psi_R^a\right). \tag{34}$$

Note that the corresponding set in $d = 3+1$ dimensions would be smaller as the γ_4 terms would be constrained by a larger Lorentz symmetry.

In a (partially) bosonized language after a Hubbard-Stratonovich transformation, we encounter six boson-fermion interactions: The first one corresponding to Eq. (31) couples the fermions to a scalar boson (scalar with respect to $\mathcal{P}$ parity), the second and the third (32) to a pseudo-scalar boson, the fourth and the fifth (33) to a vector boson, and the sixth (34) to a pseudo-vector boson. Further bosonic structures in the flavor-singlet channels appear in the corresponding Fierz transforms of Eqs. (31)-(34).

We expect that a general model based on these interactions exhibits a rich phase structure, being controlled by the relative strength of the various interaction channels. Aiming at an analogue of the electroweak phase transition, we focus in this work on the $\mathcal{P}$ parity-conserving and Lorentz-invariant condensation channel, parameterized in terms of the first interaction term. This channel is also invariant under $\mathcal{T}$ time reversal, whereas the boson transforms into its complex conjugate under $\mathcal{C}$ charge conjugation. Moreover, we confine ourselves to the case of $N_R = 1$ right-handed fermion flavor and $N_L \geq 1$ left-handed fermion flavors, allowing for a left/right asymmetry similar to the standard model of particle physics (where $N_L = 2$). The microscopic action of our model in the purely fermionic language then reads

$$S_{\text{4-fermi}} = \int d^3x \big\{ \bar{\psi}_L^a i\slashed{\partial}\psi_L^a + \bar{\psi}_R i\slashed{\partial}\psi_R \\ + 2\lambda\left(\bar{\psi}_L^a \psi_R\right)\left(\bar{\psi}_R \psi_L^a\right)\big\}. \tag{35}$$

Via Hubbard-Stratonovich transformation, we obtain the equivalent Yukawa action

$$S_{\text{Yuk}} = \int d^3x \big\{ \frac{1}{2\lambda}\phi^{a\dagger}\phi^a + \bar{\psi}_L^a i\slashed{\partial}\psi_L^a + \bar{\psi}_R i\slashed{\partial}\psi_R \\ + \phi^{a\dagger}\bar{\psi}_R \psi_L^a - \phi^a \bar{\psi}_L^a \psi_R\big\}, \tag{36}$$

where the complex scalar ϕ^a serves as an auxiliary field. The purely fermionic model can be recovered by use of the algebraic equations of motion for ϕ^a and $\phi^{a\dagger}$,

$$\phi^a = -2\lambda\bar{\psi}_R \psi_L^a, \quad \phi^{a\dagger} = 2\lambda\bar{\psi}_L^a \psi_R. \tag{37}$$

Here, we can read off the transformation properties of the scalar field under the chiral symmetry,

$$\phi^a \mapsto U_L^{ab}\phi^b U_R^\dagger, \quad \phi^{a\dagger} \mapsto U_R \phi^{b\dagger}(U_L^\dagger)^{ba}. \tag{38}$$

The composite scalar field ϕ^a represents an order parameter for an order-disorder transition. As long as ϕ^a has a vanishing expectation value, the system is in the symmetric phase with full chiral $U(N_L)_L \times U(1)_R$ symmetry; the fermions are massless, whereas the scalars are generically massive as determined by the symmetry-preserving effective potential for the scalars. If ϕ^a acquires a vacuum expectation value the chiral $SU(N_L)$ factor is broken down to a residual $SU(N_L - 1)$ symmetry. In addition, the axial $U(1)_A$ is broken, whereas the charge-conserving vector $U(1)_V$ is preserved. In the broken phase, the spectrum consists of one massive Dirac fermion, one massive bosonic radial mode, $N_L - 1$ massless left-handed Weyl fermions and $2N_L - 1$ massless Goldstone bosons.

Near the phase transition, we expect the order-parameter fluctuations to dominate the critical behavior of the system. Universality suggests that the degrees of

freedom parameterized by the action (36) are sufficient to quantify the critical behavior of this transition, independently of the presence of further microscopic fermionic interactions of Eqs. (31)-(34). Concentrating on the action (36), we observe that the purely scalar sector, i.e., the scalar mass term, has a larger symmetry group of $O(2N_L)$-type. It is therefore instructive to compare the critical behavior of our fermionic model with that of a standard scalar $O(2N_L)$ model which is known to undergo a second order phase transition associated with a Wilson-Fisher fixed point. Differences in the corresponding critical behaviors can then fully be attributed to fermionic fluctuations near the phase transition.

III. EFFECTIVE AVERAGE ACTION AND RG FLOW

Integrating out fluctuations momentum shell by momentum shell near the phase transition described above, the effective action (effective Wilson-type Hamiltonian) is expected to acquire all possible operators of mixed scalar and fermionic nature compatible with the symmetries of the action (36). In the present work, we constrain the flow of this action functional to lie in the subspace of the full theory space spanned by the following ansatz valid at an RG scale k:

$$\Gamma_k = \int d^d x \Big\{ Z_{L,k} \bar{\psi}_L^a i\slashed{\partial}\psi_L^a + Z_{R,k}\bar{\psi}_R i\slashed{\partial}\psi_R$$
$$+ Z_{\phi,k}\left(\partial_\mu \phi^{a\dagger}\right)\left(\partial^\mu \phi^a\right) + U_k(\phi^{a\dagger}\phi^a)$$
$$+ \bar{h}_k \bar{\psi}_R \phi^{a\dagger}\psi_L^a - \bar{h}_k\bar{\psi}_L^a \phi^a \psi_R \Big\}. \tag{39}$$

The fermion fields ψ_L^a and ψ_R have standard kinetic terms but can pick up different wave function renormalizations $Z_{L,k}$ and $Z_{R,k}$. The index a runs from 1 to N_L. The bosonic sector involves a standard kinetic term with wave function renormalization $Z_{\phi,k}$ and an effective potential $U_k(\rho)$, where $\rho = \phi^{a\dagger}\phi^a$. It is sometimes useful, to express the complex scalar field in terms of a real field basis by defining

$$\phi^a = \frac{1}{\sqrt{2}}(\phi_1^a + i\phi_2^a), \quad \phi^{a\dagger} = \frac{1}{\sqrt{2}}(\phi_1^a - i\phi_2^a), \tag{40}$$

where $\phi_1^a, \phi_2^a \in \mathbb{R}$. All parameters in the effective average action are understood to be scale dependent, which is indicated by the momentum-scale index k. The scale dependence is governed by the Wetterich equation, which allows for a nonperturbative construction of quantum field theory in terms of the effective average action Γ_k [16]:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\text{STr}\left\{\left[\Gamma_k^{(2)}[\Phi] + R_k\right]^{-1}(\partial_t R_k)\right\}. \tag{41}$$

Here, $\Gamma_k^{(2)}[\Phi]$ is the second functional derivative with respect to the field Φ, the latter representing a collective

field variable for all bosonic or fermionic degrees of freedom, and R_k denotes a momentum-dependent regulator function that suppresses IR modes below a momentum scale k. The solution to the Wetterich equation provides an RG trajectory in theory space, interpolating between the bare action S_{Yuk} to be quantized $\Gamma_{k\to\Lambda} \to S_{\text{Yuk}}$ and the full quantum effective action $\Gamma = \Gamma_{k\to 0}$, being the generating functional of 1PI correlation functions. For reviews, see e.g., [17].

The ansatz (39), in fact, represents the next-to-leading order in a systematic derivative expansion of the effective potential in the scalar sector and a leading-order vertex expansion in the fermionic sector. (The leading-order derivative expansion is obtained by setting all wave function renormalizations $Z_{(\phi,L,R),k} = \text{const.}$) This expansion can consistently be extended to higher orders and thus defines a legitimate and controllable nonperturbative approximation scheme. It has indeed proved its quantitative reliability already in a number of examples involving Yukawa sectors [6, 20, 21, 22].

In order to fix the standard RG invariance of field rescalings, we define the renormalized fields as

$$\tilde{\phi} = Z_{\phi,k}^{1/2}\phi, \quad \tilde{\psi}_{L/R} = Z_{L/R,k}^{1/2}\psi_{L,R}. \tag{42}$$

For the search for a fixed point where the system is scale invariant, it is useful to introduce dimensionless renormalized quantities. In order to display the dimension dependence, we perform the analysis in d spacetime dimensions, where the dimensionless renormalized field, Yukawa coupling and scalar potential read

$$\tilde{\rho} = Z_{\phi,k}k^{2-d}\rho, \tag{43}$$
$$h_k^2 = Z_{\phi,k}^{-1}Z_{L,k}^{-1}Z_{R,k}^{-1}k^{d-4}\bar{h}_k^2, \tag{44}$$
$$u_k(\tilde{\rho}) = k^{-d}U_k(\rho)|_{\rho=k^{d-2}\tilde{\rho}/Z_{\phi,k}}. \tag{45}$$

The flow of the wave function renormalizations $Z_{\phi,k}, Z_{L,k}$ and $Z_{R,k}$ can be expressed in terms of scale-dependent anomalous dimensions

$$\eta_\phi = -\partial_t \ln Z_{\phi,k}, \quad \eta_{L/R} = -\partial_t \ln Z_{L/R,k}. \tag{46}$$

With these preliminaries, we can compute the flow of the effective potential (see App. A),

$$\partial_t u_k = -du_k + \tilde{\rho}(d - 2 + \eta_\phi)u_k' + 2v_d\Big[(2N_L - 1)l_0^d(u_k')$$
$$+ l_0^d(u_k' + 2\tilde{\rho}u_k'') - d_\gamma(N_L - 1)l_{0,L}^{(F)d}(0)$$
$$- d_\gamma l_{0,L}^{(F)d}(h_k^2\tilde{\rho}) - d_\gamma l_{0,R}^{(F)d}(h_k^2\tilde{\rho})\Big], \tag{47}$$

where $v_d^{-1} = 2^{d+1}\pi^{d/2}\Gamma(d/2)$, $d_\gamma = 4$ is the dimension of the γ matrices, and the threshold functions,

$$l_n^d(\omega) = \frac{2(\delta_{n,0} + n)}{d}\left(1 - \frac{\eta_\phi}{d+2}\right)\frac{1}{(1+\omega)^{n+1}},$$
$$l_{n,L/R}^{(F)d}(\omega) = \frac{2(\delta_{n,0} + n)}{d}\left(1 - \frac{\eta_{L/R}}{d+1}\right)\frac{1}{(1+\omega)^{n+1}}. \tag{48}$$

encode the information about a possible decoupling of massive modes. For the momentum regularization, we have used a linear regulator function R_k here which is optimized for the derivative truncation [23].

Whereas the flow of the Yukawa coupling is unambiguous in the symmetric regime, the Goldstone and radial modes can generally develop different couplings in the broken regime. Here, we concentrate on the Goldstone-mode Yukawa coupling to the fermions, as the radial mode becomes massive and decouples in the broken regime. In both phases, the flow of the Yukawa coupling

can be written as (see App. A)

$$\partial_t h_k^2 = (\eta_\phi + \eta_L + \eta_R + d - 4)h_k^2 \tag{49}$$
$$-8v_d h_k^4 \kappa_k u_k'' l_{111}^{(FB)d}(\kappa_k h_k^2, u_k' + 2\kappa_k u_k'', u_k'),$$

where the scalar-potential terms have to be evaluated on the k-dependent minimum of the potential $\tilde{\rho}_{\min} \equiv \kappa_k$. In the symmetric regime, we have, of course, $\kappa_k = 0$. The threshold function occurring in Eq. (49) reads for the linear regulator:

$$l_{n_1,n_2,n_3}^{(FB)d}(\omega_1, \omega_2, \omega_3) = \frac{2}{d} \frac{1}{(1+\omega_1)^{n_1}(1+\omega_2)^{n_2}(1+\omega_3)^{n_3}} \times$$
$$\left[\frac{n_1}{1+\omega_1} \left(1 - \frac{\frac{1}{2}(\eta_L + \eta_R)}{d+1}\right) + \frac{n_2}{1+\omega_2}\left(1 - \frac{\eta_\phi}{d+2}\right) + \frac{n_3}{1+\omega_3}\left(1 - \frac{\eta_\phi}{d+2}\right) \right].$$

Setting the anomalous dimensions to zero defines the leading-order derivative expansion. At next-to-leading order, it is important to distinguish between $Z_{L,k}$ and $Z_{R,k}$ as they acquire different loop contributions, see below. The flows of the anomalous dimensions read (see App. A)

$$\eta_\phi = \frac{16v_d}{d}u_k''^2 \kappa_k m_{22}^d(u_k' + 2\kappa_k u_k'', u_k') + \frac{8v_d d_\gamma}{d}\left[\kappa_k h_k^4 m_2^{(F)d}(\kappa_k h_k^2) + h_k^2 m_4^{(F)d}(\kappa_k h_k^2)\right],$$
$$\eta_L = \frac{8v_d}{d}h_k^2\left[m_{12}^{(FB)d}(h_k^2\kappa_k, u_k' + 2\kappa_k u_k'') + m_{12}^{(FB)d}(h_k^2\kappa_k, u_k')\right], \tag{50}$$
$$\eta_R = \frac{8v_d}{d}h_k^2\left[m_{12}^{(FB)d}(h_k^2\kappa_k, u_k' + 2\kappa_k u_k'') + m_{12}^{(FB)d}(h_k^2\kappa_k, u_k') + 2(N_L - 1)m_{12}^{(FB)d}(0, u_k')\right],$$

where the potential is again evaluated at $\tilde{\rho} = \kappa_k$. Here, we have also introduced the corresponding threshold functions for the linear regulator

$$m_{n_1,n_2}^d(\omega_1, \omega_2) = \frac{1}{(1+\omega)^{n_1}(1+\omega)^{n_2}},$$
$$m_2^{(F)d}(\omega) = \frac{1}{(1+\omega)^4},$$
$$m_4^{(F)d}(\omega) = \frac{1}{(1+\omega)^4} + \frac{1-\eta_\psi}{d-2}\frac{1}{(1+\omega)^3}$$
$$-\left(\frac{1-\eta_\psi}{2d-4} + \frac{1}{4}\right)\frac{1}{(1+\omega)^2},$$
$$m_{n_1,n_2}^{(FB)d}(\omega_1, \omega_2) = \left(1 - \frac{\eta_\phi}{d+1}\right)\frac{1}{(1+\omega_1)^{n_1}(1+\omega_2)^{n_2}}, \tag{51}$$

and $\eta_\psi := \frac{1}{2}(\eta_R + \eta_L)$.

IV. FIXED POINTS AND CRITICAL EXPONENTS

The Wetterich equation provides us with the flow of the generalized couplings $\partial_t g_i = \beta_i(g_1, g_2, \dots)$ of our trunca-

tion, where the g_i correspond to the Yukawa coupling and, e.g., the expansion coefficients of the scalar potential, etc. A fixed point g^* is defined by

$$\forall i: \quad \beta_i(g_1^*, g_2^*, \dots) = 0. \tag{52}$$

If the fixed point separates the disordered from an ordered phase, it is a candidate for a second-order phase transition. For the investigation of the critical properties of the theory near this transition, we consider the fixed-point regime, where the flow can be linearized around the fixed point,

$$\partial_t g_i = B_i^{\ j}(g_j^* - g_j) + \dots, \quad B_i^{\ j} = \left.\frac{\partial \beta_i}{\partial g_j}\right|_{g=g^*}. \tag{53}$$

Let us denote the eigenvalues of the stability matrix $B_i^{\ j}$ by ω_I. The index I labels the order of the eigenvalues according to their real part, starting with the smallest one which we call ω_0. Negative ω_I correspond to RG relevant directions for which an IR (UV) fixed point is repulsive (attractive). In analogy to the notion of critical phenomena in $O(N)$-models, we define $\nu = -1/\omega_0$, characterizing the critical exponent of the correlation length near the critical temperature. This is indeed justified, as the largest critical exponent is associated with the

strongest RG relevant direction, being in turn related to the distance from the critical temperature. Thus, ν, in fact, corresponds to the standard correlation-length exponent. The subleading exponent is traditionally called $\omega = \omega_1$.

In order to analyze the fixed point structure of this model, we will distinguish two different regimes of the system namely the symmetric regime (SYM), where the vacuum expectation value (vev) of the boson field is zero, and the regime of spontaneously broken symmetry (SSB), where it is nonzero. As boson and fermion fluctuations generically contribute with opposite sign, the existence of a fixed point requires a balancing between both contributions together with potential dimensional scaling terms (such as, e.g., the first term on the right-hand side of Eq. (49)). We observe that this balancing is indeed possible in both regimes, depending on the number of left-handed fermion flavors. The origin of this flavor-number dependence is illustrated for the running of the scalar mass term or vacuum expectation value in Fig. 1. Whereas the scalar loop carries a weight $\sim N_L$ as all
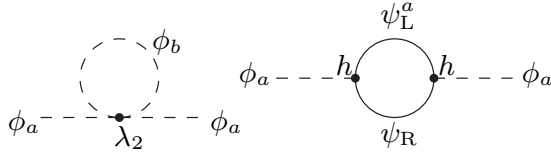

FIG. 1: Loop contributions to the renormalization flow of the mass or the vev. The left loop involves only inner boson lines. The vertex λ_2 allows for a coupling between all available boson components. This implies a linear dependence on N_L for the renormalization of the boson contribution, see below. On the right panel, we depict the corresponding fermion loop contribution. The incoming boson ϕ_a fully determines the structure of the fermion loop and does not allow for other left-handed inner fermions than ψ_L^a, inhibiting an N_L dependence of the algebraic weight of this loop.

scalar degrees of freedom can contribute, the left/right asymmetry structure leads to a weight $\sim \mathcal{O}(1)$ for the fermion loops in Fig. 1. In the following, we analyze the different regimes in detail.

A. The symmetric regime

Here and in the following, we drop the subscript k indicating the scale dependence of the running couplings for simplicity. The presence of this scale dependence will be implicitly understood. Also we restrict our investigations to $d = 3$ from now on.

In the symmetric regime, we employ the following ex-

pansion of the effective potential u

$$u \equiv u_k = \sum_{n=1}^{N_p} \frac{\lambda_n}{n!} \tilde{\rho}^n = m^2 \tilde{\rho} + \frac{\lambda_2}{2!} \tilde{\rho}^2 + \frac{\lambda_3}{3!} \tilde{\rho}^3 + ...(54)$$

($m^2 \equiv \lambda_1$). The minimum is assumed to be at $\tilde{\rho} = 0$, implying $\kappa = 0$. This also implies that $m^2 \geq 0$. This expansion is plugged into the flow equations for the effective potential, the Yukawa coupling and the anomalous dimensions. An expansion of the whole flow equation in terms of $\tilde{\rho}$ yields the flows of the different running couplings λ_n.

First, we write down the explicit expression for the Yukawa coupling, which in the SYM regime reduces to

$$\partial_t h^2 = (\eta_\phi + \eta_L + \eta_R - 1)h^2. \tag{55}$$

This tells us that an interacting fixed point ($h \neq 0$) can only occur if

$$\eta_\phi + \eta_L + \eta_R = 1. \tag{56}$$

Conversely, if a fixed point exists in the SYM regime, the sum rule (56) has to be satisfied by the anomalous dimensions. This statement holds exactly in the present truncation but may receive corrections from higher orders. We comment further on the relevance of this sum rule in the conclusions. The expressions for the anomalous dimensions read

$$\eta_\phi = \frac{h^2}{3\pi^2}(5 - \eta_L - \eta_R), \tag{57}$$

$$\eta_L = \frac{h^2}{6\pi^2}(4 - \eta_L)\frac{1}{(1+m^2)^2}, \tag{58}$$

$$\eta_R = N_L \frac{h^2}{6\pi^2}(4 - \eta_L)\frac{1}{(1+m^2)^2}. \tag{59}$$

This is a linear system of equations which can be solved analytically. Its solution expresses the anomalous dimensions in terms of the couplings h^2 and m^2,

$$\eta_\phi = \frac{2h^2(2h^2(N_L + 1) - 15\pi^2(1+m^2)^2)}{h^4(N_L + 1) - 18\pi^4(1+m^2)^2}, \tag{60}$$

$$\eta_L = \frac{h^2(5h^2 - 12\pi^2)}{h^4(N_L + 1) - 18\pi^4(1+m^2)^2}, \tag{61}$$

$$\eta_R = N_L \frac{h^2(5h^2 - 12\pi^2)}{h^4(N_L + 1) - 18\pi^4(1+m^2)^2}. \tag{62}$$

These expressions can be plugged into the sum rule (56), resulting in a conditional fixed point for h^2 depending on the size of m^2, which reads

$$h^2_{\text{cond}} = \frac{3\pi^2}{8(N_L+1)}\left\{7 + 5m(2+m) + 2N_L - \sqrt{33 + m(2+m)(54 + 25m(2+m)) + 12N_L + 4m(2+m)N_L + 4N_L^2}\right\}.$$

For another solution with a positive root, we have not been able to identify a true fixed point of the full system by numerical means. Hence, this solution is ignored in the following. The solution with the negative root, however, does give a fixed point and will be analyzed in the following. This solution is positive for all $m^2 > 0$ and monotonously increasing. It ranges between $h^2(m^2 = 0) = \frac{3\pi^2}{8}\left(7 + 2N_L - \sqrt{33 + 4N_L(3 + N_L)}\right)/(N_L + 1)$ and $h^2(m^2 = \infty) = \frac{3\pi^2}{5}$, which is very convenient because it ensures that the fixed point value for h^2 is bounded from above and from below in a very narrow window for all m^2. A true fixed point of the system requires fixed points for all scalar couplings ($m^2(= \lambda_1), \lambda_2, \lambda_3, ...$). The flow equation of a coupling λ_n is always a function of the lower order couplings from the effective potential up to λ_{n+1}, i.e.

$$\partial_t \lambda_n = f_n(h^2, \lambda_1, ..., \lambda_{n+1}). \tag{63}$$

Inserting the conditional fixed point for the Yukawa couplings (63) into these flows leaves us with the problem of searching for a scalar fixed-point potential. This problem is familiar from scalar theories where the Wilson-Fisher fixed point follows from an equation similar to Eq. (63) (with $h^2 = 0$, of course). We solve the fixed-point equations $\partial_t \lambda_n = 0$ approximately by a polynomial expansion of the potential up to some finite order $n \leq n_{max}$. Dropping the higher-order couplings $\lambda_{n>n_{max}} = 0$, the resulting system of fixed-point equations can be solved explicitly. We find suitable fixed-point solutions in the symmetric regime for $N_L \in \{1, 2\}$. The non-universal fixed-point values as well as the universal values for the anomalous dimensions at the fixed point and the first two critical exponents can be read off from Table II. For these results, we have expanded the effective potential up to λ_6 at next-to-leading order in the derivative expansion and computed the corresponding stability matrix, c.f. Eq. (53), including the Yukawa coupling flow. We would like to stress that the fixed-point equations in the present case are technically much more involved in comparison with those of scalar O(N) models in a similar approximation, as the insertion of the conditional Yukawa coupling fixed point introduces a much higher degree of nonlinearity.

TABLE II: Fixed-point values and critical exponents in the SYM regime. Fixed points corresponding to a second-order phase transition of the system exist in this regime only for $N_L = 1, 2$.

N_L	h_*^2	m_*^2	λ_2^*	η_ϕ^*	η_L^*	η_R^*	ν	ω
1	4.496	0.326	5.099	0.716	0.142	0.142	1.132	0.786
2	3.364	0.104	3.643	0.512	0.162	0.325	1.100	0.809

We emphasize that a corresponding scalar O($2N_L$) model does not exhibit a fixed-point potential in the SYM regime but only in the SSB regime. We conclude that the

nature of the phase transition and the corresponding critical behavior is characteristic for our fermionic model. In particular for small N_L, the fermionic fluctuations contribute with a comparatively large weight to the critical behavior, as discussed in Fig. 1.

In order to estimate the error on our results arising from the polynomial expansion of the effective potential, we study the convergence of the fixed-point values and of the critical exponents as a function of increasing truncation order for $N_L = 2$. Figure 2 displays our results for a truncation beyond order $\rho^n \sim \phi^{2n}$ for $n = 2, 4, 6, 7$. All quantities show a satisfactory convergence with a variation on the 1% level among the highest-order truncations.
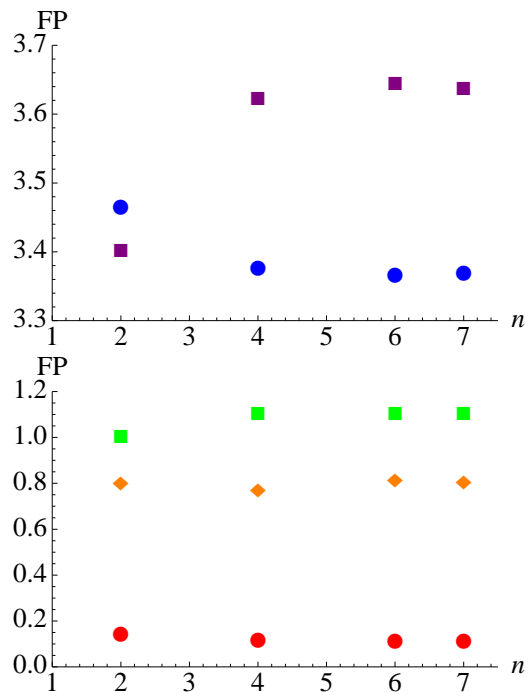

FIG. 2: Error estimate for $N_L = 2$: fixed-point values $h_*^2, m_*^2, \lambda_2^*$ and critical exponents ν, ω as a function of the highest-order term n in the ρ expansion of the effective potential. Left panel: h_*^2 (blue dots), λ_2^* (purple squares). Right panel: m_*^2 (red dots), ν (green squares), ω (orange diamonds).

Of course, the full truncation error introduced by the derivative expansion is much harder to determine and depends on the specific quantity. By analogy with the bosonic O(N) models, we expect the leading critical exponent to be our most accurate quantity. On the same truncation level, the critical exponent ν in O(N) models agrees with the best known value already on the 3% level, see [19]. The subleading exponents as well as the anomalous dimensions usually are less well approximated to this order of the derivative expansion and require more refined techniques for a better resolution of the momentum dependence, e.g., as suggested in [19, 24].

B. The regime of spontaneous symmetry breaking (SSB)

For increasing left-handed fermion number, the scalar fixed-point potential must eventually lie in the regime of spontaneous symmetry breaking. This is already obvious from the structure of the flow equations: for large N_L, the Goldstone-like fluctuation modes dominate the flow of the effective potential (47) (the N_L-dependent fermionic contribution in Eq. (47) is field independent and can be dropped). Therefore, the potential flow of our model approaches that of an $O(2N_L)$ model in the limit $N_L \to \infty$. The latter is known to exhibit a Wilson-Fisher fixed-point potential in the SSB regime with a nonzero $\kappa^* > 0$.

We observe the transition of the fixed-point potential from the symmetric to the SSB regime already near $N_L = 3$. The properties of the Wilson-Fisher fixed point in the analogous bosonic model are, of course, quantitatively modified by the presence of fermionic fluctuations, but its basic characteristics are otherwise left intact. Based on these simple observations, we continue with an analysis of the fixed-point structure in the SSB regime in the remainder of this section. For the SSB phase where the effective potential u is minimal at a nonzero value $\kappa > 0$, we use the expansion (dropping the subscript k again for simplicity)

$$u = \sum_{n=2}^{N_P} \frac{\lambda_n}{n!}(\tilde\rho - \kappa)^n = \frac{\lambda_2}{2!}(\tilde\rho-\kappa)^2 + \frac{\lambda_3}{3!}(\tilde\rho-\kappa)^3 + \dots \quad (64)$$

For the flow of κ, we use the fact that the first derivative of u vanishes at the minimum, $u'(\kappa) = 0$. This implies

$$0 = \partial_t u'(\kappa) = \partial_t u'(\tilde\rho)|_{\tilde\rho=\kappa} + (\partial_t\kappa)u''(\kappa)$$
$$\Rightarrow \partial_t\kappa = -\frac{1}{u''(\kappa)}\partial_t u'(\tilde\rho)|_{\tilde\rho=\kappa}. \quad (65)$$

The flow equations in the SSB regime are more involved due to additional loop contributions which arise from the coupling to the vev. This higher degree of nonlinearity inhibits a simple analytical study of the fixed-point structure at NLO in the derivative expansion. Instead, we use an iterative method, starting at the Wilson-Fisher fixed point for the analogous $O(2N_L)$ model. This fixed point can be obtained in our system by setting the Yukawa coupling to zero, $h^2 = 0$.[2] Starting at this Wilson-Fisher fixed point of the reduced scalar system, we can obtain a fixed point of the full chiral Yukawa system by numerical iteration. This confirms that the presence of the fermions generically shifts the scalar fixed-point values

only slightly. However, our chiral Yukawa system represents a different universality class, and so the critical exponents and the anomalous dimensions are special to our system. Numerical results are displayed in Figs. 3-5.
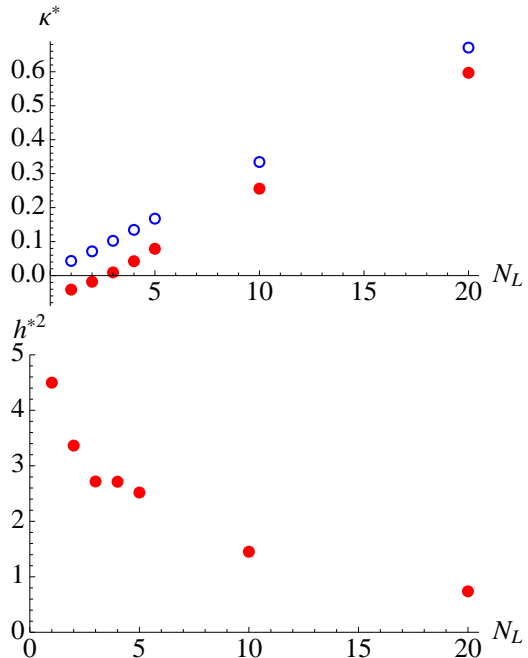

FIG. 3: Nonuniversal fixed point values in the SSB regime. Left panel: the potential minimum κ^* in our $U(N_L)_L \otimes U(1)_R$ model (red dots) are compared with those of the $O(2N_L)$ model (blue circles). The negative κ^* values for $N_L < 3$ are particular to our model and correspond to the fact that the fixed-point potential is in the symmetric regime. This transition near $N_L \lesssim 3$ also causes a kink in the N_L dependence of h^{*2} (right panel).

In Fig. 3, the fixed-point values of κ and h^2 are plotted for different N_L. Note that κ^* does not satisfy the constraint $\kappa \geq 0$ for $N_L < 3$; these negative values are therefore unphysical. Of course, this negative branch corresponds to the fixed-point scenario in the symmetric regime discussed in the previous section. As expected, the fixed-point values of our $U(N_L)_L \otimes U(1)_R$ model (red dots) change only slightly in comparison with the analogous $O(2N_L)$ model (blue circles). Both models were investigated with anomalous dimensions and expanded up to order $\sim \phi^{12}$ in the effective potential. It should be stressed that the fixed-point values themselves are nonuniversal quantities depending on the details of the regulator.

By contrast, the fixed-point values of the anomalous dimensions as plotted in Fig. 4 are universal (even though slight regulator dependencies can be induced by the truncation). The left panel shows η_ϕ^* for our $U(N_L)_L \otimes U(1)_R$ model (red dots) in comparison with the analogous $O(2N_L)$ model (blue circles). Whereas η_ϕ^* in both models approaches a common value for large N_L, we observe larger differences for smaller N_L which can

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[2] This offers also the possibility of a cross check: a comparison of our $O(2N_L)$-model results to the results which are given in [19] reveals a very satisfactory precision. The remaining minor deviations can fully be attributed to the fact that we use a simple NLO-derivative expansion truncated at λ_6.

directly be attributed to the fermionic loop contributions. The fermion anomalous dimensions η_L^* and η_R^* are shown in the right panel (purple diamonds and green triangles, respectively). For $N_L = 1$, both anomalous dimensions agree as this corresponds to the left/right symmetric point. For larger N_L, η_R^* becomes significantly larger, as the massless fermion and boson degrees of freedom contribute to the loop diagrams with a weight $\sim N_L$.
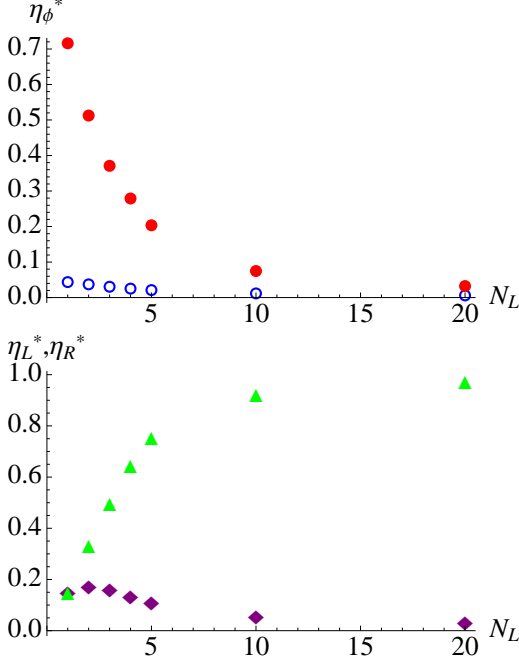

FIG. 4: Left: η_ϕ^* in the $U(N_L)_L \otimes U(1)_R$ model (red dots) in comparison with that of the analogous $O(2N_L)$ model (blue circles). Right: η_L^* (purple diamonds) and η_R^* (green triangles) in the chiral Yukawa model.

The critical exponents ν and ω are shown in Fig. 5. Again, the red dots represent the values for our chiral $U(N_L)_L \otimes U(1)_R$ model, which is compared with those of the analogous $O(2N_L)$ model (blue circles). The results of both models approach each other for large N_L as expected but show sizable deviations at smaller N_L. We observe a rapid change of the critical exponents near $N_L \simeq 3$, where the effective fixed point potential changes from the symmetric to the SSB regime. This strong variation is rather natural, as the structure of the fixed-point equations varies significantly across this transition.

We believe that especially the results for the universal fixed-point anomalous dimensions and critical exponents as summarized in Tab. III can provide for first benchmarks for these new universality classes. For any other nonperturbative tool for chiral fermions near the symmetry-breaking phase transition, these universality classes can serve as a useful testing ground. As in the symmetric regime discussed above, we consider our results for the exponent ν to be reliable on the few-percent level, whereas the subleading exponent as well as the
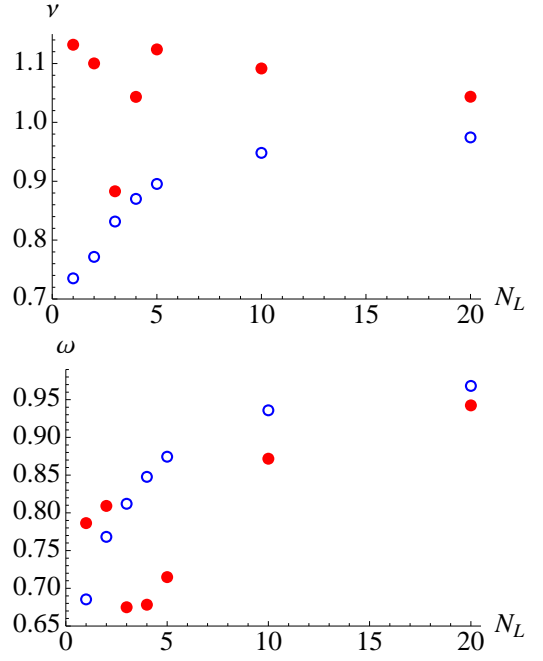

FIG. 5: Critical exponents as a function of N_L in the chiral $U(N_L)_L \otimes U(1)_R$ model (red dots) in comparison with those of the analogous $O(2N_L)$ model (blue circles). The rapid change of the exponents occurs near $N_L \simeq 3$, where the fixed-point potential shows a transition from the symmetric to the SSB regime.

TABLE III: Fixed point values and critical exponents in the SSB regime.

N_L	h_*^2	κ_*	λ_2^*	η_ϕ^*	η_L^*	η_R^*	ν	ω
3	2.718	0.009	2.967	0.371	0.154	0.487	0.883	0.675
4	2.713	0.042	2.954	0.279	0.125	0.637	1.043	0.678
5	2.519	0.079	2.717	0.204	0.100	0.746	1.124	0.715
10	1.452	0.256	1.506	0.075	0.046	0.913	1.092	0.872
20	0.739	0.597	0.752	0.032	0.022	0.963	1.043	0.942
50	0.296	1.612	0.298	0.012	0.009	0.986	1.017	0.978
100	0.148	3.301	0.149	0.006	0.004	0.993	1.008	0.989

anomalous dimensions might be somewhat less accurate.

V. CONCLUSIONS

We have investigated the critical behavior of three-dimensional relativistic fermion models with a $U(N_L)_L \times U(1)_R$ chiral symmetry. We have designed this class of models to exhibit similarities to the Higgs-Yukawa sector of the standard model of particle physics. As a fundament of this model building, we have classified all possible four-fermion interaction terms invariant under this chiral symmetry, and also determined the corresponding discrete symmetries. We have identified a scalar parity-conserving channel similar to the standard-model Higgs scalar. For sufficiently strong correlations in this channel,

a second-order phase transition into the chiral-symmetry broken phase can occur which is a 3d analog of the electroweak phase transition. Using the functional RG, we have computed the critical behavior of this phase transition in terms of the critical correlation-length exponent ν, the subleading exponent ω, and the fermion and scalar anomalous dimensions as a function of $N_{\mathrm{L}} \geq 1$.

Whereas the standard model is defined with a fundamental Higgs scalar, we have here started with only fundamental fermion degrees of freedom. The resulting Higgs field arises as a scalar bi-fermionic composite upon strong fermionic correlations. Therefore, the relation between our original fermion model and the resulting Yukawa system is similar to that between top-quark condensation models [25] and the standard-model Higgs sector. From a more general RG perspective, however, the purely fermionic models can anyway be viewed as just a special case of the more general Yukawa models [26, 27] supplemented with nonuniversal compositeness conditions. As long as the compositeness scale in the deep UV remains unresolved, there is no real difference between the purely fermionic or the Yukawa-model language.

For our quantitative results, we have used a consistent and systematic expansion scheme of the effective action in terms of a nonperturbative derivative expansion. Whereas there is an extensive body of circumstantial evidence in the literature that this expansion is suitably adjusted to the relevant degrees of freedom of Yukawa systems, a practical test for convergence is problematic in the present case. This is because the leading-order of the expansion (defined by setting all $\eta_{\phi,\mathrm{L,R}} = 0$) does not support the desired fixed point. The latter becomes visible only from next-to-leading order on, due to the structure of the Yukawa flow. Hence, a straightforward convergence test in principle requires a NNLO calculation. Instead, the reliability of the results can also be verified indirectly: first of all, the derivative expansion is based on the implicit assumption that momentum dependencies of operators do not grow large. This includes the kinetic terms, such that self-consistency of the derivative expansion requires that the anomalous dimensions satisfy $\eta_{\phi,\mathrm{L,R}} \lesssim 1$, as is the case in our calculations. Second, our models can always be compared to the purely bosonic limit of scalar $\mathrm{O}(N)$ models where the quantitative reliability of the derivative expansion has been verified to a high level of significance. Third, we observe very good convergence properties of the polynomial expansion of the effective potential which is again a strong signature of self-consistency.

From our classification of all fermionic interaction terms compatible with the required symmetries, it is clear that the Higgs-like condensation channel is not the only possible channel. Aside from vector-like channels, there are two further pseudo-scalar channels, cf. Eq. (32), and further scalar and pseudo-scalar channels in the flavor-singlet Fierz transforms of Eqs. (31)-(34). In fact, the present analysis is a restricted study of a particular condensation process. We expect that the phase diagram of the general model is much more involved and might exhibit a variety of possible phases and corresponding transitions. This phase diagram is parameterized by up to 6 independent couplings being associated with the linearly independent fermionic interactions. The calculation of the true condensation channel for a given set of initial couplings remains a challenging problem. As such a problem of competing order parameters is well known also in other systems, e.g., in the Hubbard model [28, 29, 30], the present system can serve as a rich and controllable model system.

A special feature of our model arises in the symmetric regime: here, a fixed point within our truncation implies a sum rule for the anomalous dimensions, $\eta_\phi + \eta_{\mathrm{L}} + \eta_{\mathrm{R}} = 1 \equiv 4 - d$. This sum rule is relevant, since the underlying balancing between anomalous dimensions and dimensional power-counting scaling can be a decisive feature of many other models as well. Most prominently, the asymptotic-safety scenario in quantum gravity [31] as well as in extra-dimensional Yang-Mills theories [32] requires similar sum rules to be satisfied. In contrast to these latter models, the present models for $N_{\mathrm{L}} = 1, 2$ can serve as a much simpler example for a test of this sum rule at a fixed point. A verification of this sum rule also by other nonperturbative tools can shed light on this important mechanism to generate RG fixed points.

This is another reason why we believe that the new universality classes defined by our models can serve as prototypes for studies of strongly correlated chiral fermions in general and of nonperturbative features of standard-model-like chiral symmetry breaking in particular.

Acknowledgments

APPENDIX A: FLOW EQUATIONS AND ANOMALOUS DIMENSIONS

The derivation of the flow of the dimensionless effective potential $\partial_t u_k$ (47) and of the anomalous dimensions η_{L} and η_{R} (50) are explained in the appendix of [5]. Consequently the flow of the coupling constants λ_i and the flow of the squared mass m^2 in the symmetric regime and the flow of the dimensionless squared vacuum expectation value κ in the SSB regime are the same as in [5].

For deriving the flow of the squared Yukawa coupling constant h^2 we split the bosonic field into its vev v and the deviation from the vev (the relation between the dimensionful vev v and the dimensionless squared vev κ is $\kappa = \frac{1}{2} Z_\phi k^{2-d} v^2$),

$$\phi(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1^1(p) + \mathrm{i}\phi_2^1(p) \\ \phi_1^2(p) + \mathrm{i}\phi_2^2(p) \\ \vdots \\ \phi_1^{N_\mathrm{L}}(p) + \mathrm{i}\phi_2^{N_\mathrm{L}}(p) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \\ \vdots \\ 0 \end{pmatrix} \delta(p) + \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta\phi_1^1(p) + \mathrm{i}\Delta\phi_2^1(p) \\ \Delta\phi_1^2(p) + \mathrm{i}\Delta\phi_2^2(p) \\ \vdots \\ \Delta\phi_1^{N_\mathrm{L}}(p) + \mathrm{i}\Delta\phi_2^{N_\mathrm{L}}(p) \end{pmatrix}. \tag{A1}$$

We are mainly interested in the Yukawa coupling between the fermions and the Goldstone boson. Thus we use the $\Delta\phi_2^1$ part for the projection. Using the truncation (39) and comparing with the Wetterich equation we get

$$\partial_t \bar{h}_k = -\frac{\mathrm{i}}{2} \frac{\overrightarrow{\delta}}{\delta\bar{\psi}_\mathrm{L}^1(p)} \frac{\sqrt{2}\,\overrightarrow{\delta}}{\delta\Delta\phi_2^1(p')} \mathrm{STr}\left[\tilde{\partial}_t \ln(\Gamma_k^{(2)} + R_k)\right] \frac{\overleftarrow{\delta}}{\delta\psi_\mathrm{R}(q)}\bigg|_{\substack{\psi_\mathrm{R}^a=\psi_\mathrm{L}^a=\Delta\phi=0 \\ p'=p=q=0}}. \tag{A2}$$

Next, we split $(\Gamma_k^{(2)} + R_k)$ into a propagator part $\mathcal{P}$, which contains only the vev, and a fluctuation part $\mathcal{F}$, which contains the fluctuating fields. Inserting this into Eq. (A2) the expansion of the logarithm reads

$$\ln\left(\Gamma_k^{(2)} + R_k\right) = \ln\left[\mathcal{P}\left(1 + \frac{\mathcal{F}}{\mathcal{P}}\right)\right] = \ln(\mathcal{P}) + \frac{\mathcal{F}}{\mathcal{P}} - \frac{1}{2}\left(\frac{\mathcal{F}}{\mathcal{P}}\right)^2 + \frac{1}{3}\left(\frac{\mathcal{F}}{\mathcal{P}}\right)^3 - \dots \tag{A3}$$

Only the term to third power survives the projection. Performing the matrix calculations and taking the supertrace, we get

$$\partial_t \bar{h}_k^2 = \int \frac{d^d p}{(2\pi)^d} \tilde{\partial}_t \frac{h_k^4 U_k'' v^2}{\left(Z_\phi P_\mathrm{B}(p) + U_k' + U_k'' v^2\right)\left(Z_\phi P_\mathrm{B}(p) + U_k'\right)\left(Z_\mathrm{L} Z_\mathrm{R} P_\mathrm{F}(p) + \frac{\bar{h}_k^2}{2}v^2\right)}.$$

The potential on the right-hand side is evaluated at the minimum $\frac{1}{2}v^2$. Using the threshold function and switching over to dimensionless quantities, we end up with Eq. (49).

For the derivation of the flow of η_ϕ, we use the decomposition of (A1). Again we use $\Delta\phi_2^1$ for the projection and expand the logarithm as in (A3). This time only the quadratic term survives the projection,

$$\partial_t Z_\phi = -\frac{1}{4} \frac{\partial}{\partial p^2} \frac{\delta}{\delta\Delta\phi_2^1(p)} \frac{\delta}{\delta\Delta\phi_2^1(q)} \mathrm{STr}\left[\tilde{\partial}_t \left(\frac{\mathcal{F}}{\mathcal{P}}\right)^2\right]\bigg|_{\psi=\Delta\phi=0=p=q}.$$

After performing the matrix calculations and taking the supertrace, we use $\eta_\phi = -\frac{\partial_t Z_\phi}{Z_\phi}$ and switch over to dimensionless quantities. By use of the threshold functions (51), we obtain Eqs. (50) in the main text.

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